Eur. Phys. J. C (2015) 75:138
DOI 10.1140/epjc/s10052-015-3358-0

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Pair double heavy diquark production in high energy proton–proton collisions

A. P. Martynenko[1,2,a], **A. M. Trunin**[2,3]

[1] Samara State University, Pavlov Street 1, 443011 Samara, Russia
[2] Samara State Aerospace University named after S.P. Korolyov, Moskovskoye Shosse 34, 443086 Samara, Russia
[3] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Joliot-Curie Street 6, 141980 Dubna, Russia

Received: 21 October 2014 / Accepted: 13 March 2015 / Published online: 26 March 2015

Abstract On the basis of perturbative QCD and relativistic quark model we calculate relativistic and bound state corrections in the production processes of a pair of double heavy diquarks. Relativistic factors in the production amplitude connected with the relative motion of heavy quarks and the transformation law of the bound state wave function to the reference frame of the moving S-wave diquark bound states are taken into account. For the gluon and quark propagators entering the amplitudes we use a truncated expansion in relative quark momenta up to the second order. Relativistic corrections to the quark–quark bound state wave functions in the rest frame are considered by means of a Breit-like potential. It turns out that the examined effects significantly decrease the nonrelativistic cross sections.

1 Introduction

Double heavy meson and baryon production at high energies represents an important problem of quantum chromodynamics (QCD). On the one hand the methods of nonrelativistic QCD (NRQCD) can be used in this case for a construction of production amplitudes in the leading order over the strong coupling constant α_s or for a calculation of next-to-leading order corrections [1]. On the other hand, the presence of heavy quarks gives an opportunity to explore the formation of quark bound states in these reactions on the basis of the quark model. During the last 10 years in the problem of double heavy hadron production there arises the field of research connected with double heavy quarkonium pair production. The progress was initiated by experiments of the Belle and BaBar collaborations, which measured the cross sections of pair charmonium production in e^+e^- annihilation [2–5]. The importance of such reactions for the development of theoretical methods of their investigation was demonstrated in [6–20]. An essential improvement in the theoretical description of the processes of quarkonium pair production was obtained with the assumption of a systematic account of relativistic and radiative corrections to nonrelativistic results. It was revealed that corrections of the relative motion of heavy quarks and bound state corrections essentially change the nonrelativistic calculations.

Another reaction of meson pair production was investigated recently in the pp-interaction [21]. Beginning with the start of the LHC activity, new experimental data on double heavy quarkonium production regenerated the interest in the study of quarkonium production mechanisms in hadronic collisions. The production of a $J/\psi J/\psi$ pair in the proton–proton collisions at a center-of-mass energy $\sqrt{s} = 7$ TeV has been observed with the LHCb detector. The data used for the analysis was obtained with an integrated luminosity of 37.5 pb^{-1} of pp collisions at a center-of-mass energy of $\sqrt{s} = 7$ TeV collected by the LHCb experiment in 2010. At collider energies, double charmonium production occurs for the most part through the gluon–gluon channel. The theoretical description of charmonium pair production was carried out in this case in a nonrelativistic approximation in [22–26] and with the account of relativistic corrections in [27–30]. Note that an additional uncertainty occurs in the pp-interaction due to the double parton scattering mechanism [31,32]. Along with pair quarkonium production there is interest in double heavy diquark pair production because such a process can represent a first stage of double baryon production [33–38]. The pair production of double heavy diquarks in the e^+e^-- and the $p\bar{p}$-interaction was performed in nonrelativistic QCD in [39]. An account of relativistic and bound state corrections to the cross sections in the case of e^+e^- annihilation was given in [40]. It was shown in [40] that a reliable estimate of the observed cross sections can be obtained only with a systematic account of relativistic and bound state

[a] e-mail: a.p.martynenko@samsu.ru

corrections. It is worth mentioning that the diquark (cc) and $(\bar{c}\bar{c})$ pair production at the LHC energies with subsequent formation of a tetra-quark was studied in [23,24]. Single diquark (cc) and baryon Ξ_{cc} production in different reactions was studied in [41–43]. They investigate the production of heavy quarks Q and Q' with different color and spin configuration $[n] : [n] = [^3S_1]_{\bar{\mathbf{3}}}$ (color-antitriplet diquark state), $[^1S_0]_{\mathbf{6}}$ (color-sextuplet diquark state) for the (cc) or (bb) diquark, and $(bc)_{\mathbf{3}}[^3S_1]$, $(bc)_{\mathbf{6}}[^1S_0]$, $(bc)_{\mathbf{3}}[^1S_0]$, and $(bc)_{\mathbf{6}}[^3S_1]$ for the (bc) diquark. The production of J/ψ mesons in the proton–proton collisions at $\sqrt{s} = 7$ TeV was investigated with the LHCb detector at the LHC in [44]. In this work we continue the investigation of relativistic effects in double heavy diquark pair production in proton–proton interaction at energies of the LHC on the basis of relativistic quark model. We calculate the cross section $\sigma(pp \to \mathcal{D}\bar{\mathcal{D}} + X)$ of diquark pair production in the nonrelativistic approximation and show how these results will be changed after taking account of relativistic corrections.

2 General formalism

All the models describing quarkonium production in hadronic collisions use the common basis: the factorization between the hard collision subprocess and the parton–parton collision luminosity, calculated as a convolution of the parton distribution functions (PDFs). In the collinear parton model the cross section of double heavy diquark pair production in the proton–proton collisions has the form of a convolution of the partonic cross section $d\sigma[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}]$ with the parton distribution functions of the initial protons [22,45–47]:

$$d\sigma[p + p \to D_{bc} + \bar{D}_{\bar{b}\bar{c}} + X] = \int dx_1 dx_2\, f_{g/p}(x_1, \mu) f_{g/p}(x_2, \mu)\, d\sigma[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}], \quad (1)$$

where $f_{g/p}(x, \mu)$ is the parton (gluon) distribution function (PDF) in the proton, $x_{1,2}$ are the parton momentum (longitudinal momentum) fraction from the proton, and μ is the factorization scale. Neglecting the proton mass and taking the c.m. reference frame of the initial protons with the beam along the z-axis we can present the gluon on mass-shell momenta as $k_{1,2} = x_{1,2}\frac{\sqrt{S}}{2}(1, 0, 0, \pm 1)$. $\sqrt{S}$ is the center-of-mass energy in the proton–proton collision. The range of the accessible $x_{1,2}$ depends on the rapidity interval covered by experiments. At the CM energies of the LHC the gluon–gluon contribution to the production cross section is dominant, so that we consider only gg initial states in this study. Quark–antiquark annihilation amounts to about 10 % [23,24].

According to the quasipotential approach the double heavy diquark production amplitude for the gluonic subprocess $gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}$ can be expressed as a convolution of the perturbative production amplitude of (bc) and $(\bar{b}\bar{c})$ quark and antiquark pairs $\mathcal{T}(p_1, p_2; q_1, q_2)$ and the quasipotential wave functions of the final diquarks $\Psi_{\mathcal{D}}$ [14–20,27–29]:

$$\mathcal{M}[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}](k_1, k_2, P, Q) = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3}\, \bar{\Psi}_{D_{bc}}(p, P) \bar{\Psi}_{\bar{D}_{\bar{b}\bar{c}}}(q, Q) \otimes \mathcal{T}(p_1, p_2; q_1, q_2), \quad (2)$$

where $p_{1,2}$ are four-momenta of c and b quarks, and $q_{1,2}$ are an appropriate four-momenta of $\bar{c}$ and $\bar{b}$ antiquarks. They are defined in terms of total momenta $P(Q)$ and relative momenta $p(q)$ as follows:

$$p_{1,2} = \eta_{1,2} P \pm p,\ (pP)=0;\ q_{1,2} = \eta_{1,2} Q \pm q,\ (qQ)=0, \quad \eta_{1,2} = \frac{M^2 \pm m_c^2 \mp m_b^2}{2M^2}, \quad (3)$$

where $M = M_{D_{bc}} = M_{\bar{D}_{\bar{b}\bar{c}}}$ is the double heavy diquark mass, $p = L_P(0, \mathbf{p})$ and $q = L_Q(0, \mathbf{q})$ are the relative four-momenta obtained by the Lorentz transformation of the four-vectors $(0, \mathbf{p})$ and $(0, \mathbf{q})$ to the reference frames moving with the four-momenta P and Q of the final diquarks, D_{bc} and $\bar{D}_{\bar{b}\bar{c}}$. In Eq. (2) we integrate over the relative three-momenta of quarks and antiquarks in the final state. The wave functions $\bar{\Psi}_{D_{bc}}(p, P)$ and $\bar{\Psi}_{\bar{D}_{\bar{b}\bar{c}}}(q, Q)$ determine the probability for free heavy quark $Q_1 Q_2$ and antiquark $\bar{Q}_1 \bar{Q}_2$ pairs with certain quantum numbers to transform into diquark and antidiquarks bound states (long distance matrix elements). A proof of the factorization formulas (1)–(2) deserves a special consideration. There are interactions between the initial and final hadrons connected with gluon exchanges that violate this factorization. In what follows we assume that the emission of soft and collinear gluons can be absorbed into parton distribution functions and long distance matrix elements, so that the factorization equations (1)–(2) occur. The status of a proof of the factorization in quarkonium production is presented in detail in [48–50]. A proof of factorization is essential because non-factorizing gluon contributions, for which α_s is not small, could change the numerical results. It should be mentioned that the effect induced by the radiation in the initial state was investigated in [23,24] by means of the Pythia Monte Carlo generator. It does not effect the value of the total cross section.

The parton-level differential cross section for $g + g \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}$ is expressed further through the Mandelstam variables s, t, and u:

$$s = (k_1 + k_2)^2 = (P + Q)^2 = x_1 x_2 S, \quad (4)$$

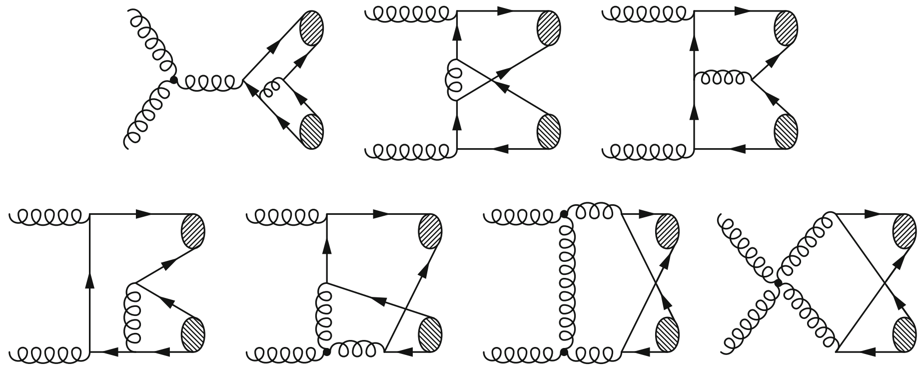

Fig. 1 The typical leading order diagrams for $gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}$ subprocess. Other diagrams can be obtained by reversing the quark lines or interchanging the initial gluons

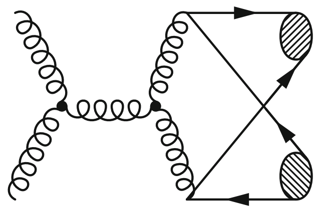

Fig. 2 The additional diagram for $gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}$ having the zero color factor

$$t = (P - k_1)^2 = (Q - k_2)^2 = M^2 - x_1\sqrt{S}(P_0 - |\mathbf{P}|\cos\phi)$$
$$= M^2 - x_1x_2S + x_2\sqrt{S}(P_0 + |\mathbf{P}|\cos\phi),$$

$$u = (P - k_2)^2 = (Q - k_1)^2 = M^2 - x_2\sqrt{S}(P_0 + |\mathbf{P}|\cos\phi)$$
$$= M^2 - x_1x_2S + x_1\sqrt{S}(P_0 - |\mathbf{P}|\cos\phi), \qquad (5)$$

where ϕ is the angle between $\mathbf{P}$ and the z-axis. The Mandelstam variables s, t, and u satisfy the relation

$$s + t + u = M^2_{D_{bc}} + M^2_{\bar{D}_{\bar{b}\bar{c}}}. \qquad (6)$$

The transverse momentum P_T of the diquark D_{bc} and its energy P_0 can be written as

$$P_T^2 = |\mathbf{P}|^2 \sin^2\phi = -t - \frac{(M^2 - t)^2}{x_1x_2S},$$
$$P_0 = \frac{x_1x_2\sqrt{S}}{x_1 + x_2} + \frac{x_1 - x_2}{x_1 + x_2}|\mathbf{P}|\cos\phi. \qquad (7)$$

In the leading order in the strong coupling constant α_s, there are 35 Feynman diagrams contributing to the gluon fusion subprocess $gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}$ of double heavy diquark pair production, which are presented in Fig. 1. One additional diagram shown in Fig. 2 appears to have a zero color factor after summation with antisymmetric color functions $\epsilon^{ijk}/\sqrt{2}$ of the final diquark states: $f^{g_1g_2e} f^{abe} T^a_{c_1c_3} T^b_{c_2c_4} \epsilon^{c_1c_2A} \epsilon^{c_3c_4B} = 0$, because we have here a convolution of antisymmetric f^{abe} and tensors symmetric over the indices a, b. f^{abc} are the structure constants of the SU(3) color group, T^a is the SU(3) generator in the fundamental representation. The diquark term $D_{Q_1Q_2}$ is used for a designation of the bound state between two heavy quarks. It can appear only if the quarks (Q_1Q_2) are initially in a color-antitriplet state. Note that the quark pair (Q_1Q_2) can couple to two irreducible color representations: $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$. But the binding of the (Q_1Q_2) pair into the diquark state can appear due to attractive color Coulomb forces only in the case of the antitriplet state. In the color-sextuplet state we have a repulsive color interaction between the quarks. It can be changed to an attractive interaction only after emission of a soft gluon. A quark pair (Q_1Q_2) production in a color sextuplet state is equivalent to the production of a pair quark–antiquark $(Q_1\bar{Q}_2)$ in a color octet state. It was shown in [26] that color octet states give a significantly smaller contribution to J/ψ meson pair production in the pp-interaction at small and intermediate momenta p_T as compared with the color singlet state. So, in this work we have analyzed only color-antitriplet contributions to diquark pair production. In view of the large volume of calculations we have used the package FeynArts [51,52] for the system Mathematica in order to obtain analytical expressions for all the diagrams and, subsequently Form [53], to evaluate their traces. Then we obtain the following result for the leading order production amplitude (2):

$$\mathcal{M}[gg \to D_{bc} + \bar{D}_{\bar{b}\bar{c}}](k_1, k_2, P, Q)$$
$$= M\pi^2\alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \mathrm{Tr}\,\mathfrak{M}, \qquad (8)$$

$$\mathfrak{M} = \bar{\Psi}^{bc}_{P,p}\gamma_\beta \bar{\Psi}^{cb}_{Q,q}\gamma_\omega \Gamma_1^{\beta\omega} + \bar{\Psi}^{cb}_{P,-p}\gamma_\beta \Gamma_2^{\beta\omega\theta}\gamma_\omega \bar{\Psi}^{bc}_{Q,-q}\gamma_\theta$$
$$+ \bar{\Psi}^{bc}_{P,p}\gamma_\beta \Gamma_3^{\beta\omega\theta}\gamma_\omega \bar{\Psi}^{cb}_{Q,q}\gamma_\theta$$
$$+ \bar{\Psi}^{bc}_{P,p}\,\hat{\varepsilon}_1 \frac{m_c - \hat{k}_1 + \hat{p}_1}{(k_1 - p_1)^2 - m_c^2}\gamma_\beta \left(\bar{\Psi}^{cb}_{Q,q}\gamma_\omega \Gamma_4^{\beta\omega} + \Gamma_5^{\beta\omega}\bar{\Psi}^{cb}_{Q,q}\gamma_\omega\right)$$
$$+ \bar{\Psi}^{cb}_{P,-p}\,\hat{\varepsilon}_1 \frac{m_b - \hat{k}_1 + \hat{p}_2}{(k_1 - p_2)^2 - m_b^2}\gamma_\beta \left(\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega \Gamma_6^{\beta\omega}\right.$$
$$\left. + \Gamma_7^{\beta\omega}\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega\right)$$
$$+ \bar{\Psi}^{bc}_{P,p}\,\hat{\varepsilon}_2 \frac{m_c - \hat{k}_2 + \hat{p}_1}{(k_2 - p_1)^2 - m_c^2}\gamma_\beta \Gamma_8^{\beta\omega}\bar{\Psi}^{cb}_{Q,q}\gamma_\omega$$
$$+ \bar{\Psi}^{cb}_{P,-p}\,\hat{\varepsilon}_2 \frac{m_b - \hat{k}_2 + \hat{p}_2}{(k_2 - p_2)^2 - m_b^2}\gamma_\beta \Gamma_9^{\beta\omega}\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega$$

$$+\bar{\Psi}^{cb}_{P,-p}\gamma_\beta \frac{m_b+\hat{k}_1-\hat{q}_2}{(k_1-q_2)^2-m_b^2}\hat{\varepsilon}_1\bar{\Psi}^{bc}_{Q,-q}\gamma_\omega\Gamma^{\beta\omega}_{10}$$
$$+\bar{\Psi}^{bc}_{P,p}\gamma_\beta \frac{m_c+\hat{k}_1-\hat{q}_1}{(k_1-q_1)^2-m_c^2}\hat{\varepsilon}_1\bar{\Psi}^{cb}_{Q,q}\gamma_\omega\Gamma^{\beta\omega}_{11}, \quad (9)$$

where $\varepsilon_{1,2}$ are polarization vectors of the initial gluons, the hat symbol means contraction of the four-vector with the Dirac gamma-matrices. A number of vertex functions Γ_i is introduced to make the entry of the amplitude (9) more compact. We explicitly extracted in (9) the normalization factors $\sqrt{2M}$ of the quasipotential bound state wave functions.

The formation of diquark states from quark and antiquark pairs, which corresponds to the first stage of double heavy baryon production, is determined in the quark model by the quasipotential wave functions $\Psi_{D_{bc}}(p,P)$ and $\Psi_{\bar{D}_{\bar{b}\bar{c}}}(q,Q)$. These wave functions are calculated initially in the meson rest frame and then transformed to the reference frames moving with the four-momenta P and Q. The law of such a transformation was derived in the Bethe–Salpeter approach in [54] and in the quasipotential method in [55]. We use the last one and obtain the following expressions for the relativistic wave functions [40]:

$$\bar{\Psi}_{D_{bc}}(p,P)=\frac{\bar{\Psi}^0_{D_{bc}}(\mathbf{p})}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}$$
$$\times\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_b(\epsilon_b(p)+m_b)}-\frac{\hat{p}}{2m_b}\right]$$
$$\times\Sigma^P(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_c(\epsilon_c(p)+m_c)}+\frac{\hat{p}}{2m_c}\right],$$
$$\bar{\Psi}_{D_{\bar{c}\bar{b}}}(q,Q)=\frac{\bar{\Psi}^0_{\bar{D}_{\bar{b}\bar{c}}}(\mathbf{q})}{\sqrt{\frac{\epsilon_c(q)}{m_c}\frac{\epsilon_c(q)+m_c}{2m_c}\frac{\epsilon_b(q)}{m_b}\frac{\epsilon_b(q)+m_b}{2m_b}}}$$
$$\times\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_c(\epsilon_c(q)+m_c)}+\frac{\hat{q}}{2m_c}\right]$$
$$\times\Sigma^Q(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_b(\epsilon_b(q)+m_b)}-\frac{\hat{q}}{2m_b}\right], \quad (10)$$

where $m_{c,b}$ are the quark masses, $\epsilon_{c,b}(p)=\sqrt{p^2+m^2_{c,b}}$, $v_1=P/M$, $v_2=Q/M$, and $\Sigma^{P,Q}$ are equal to γ_5 and $\hat{\varepsilon}_{P,Q}$ for scalar and axial-vector diquarks, respectively. The polarization vectors $\varepsilon_{P,Q}$ of the axial-vector diquarks satisfy the conditions: $(\varepsilon_P\cdot P)=0$ and $(\varepsilon_Q\cdot Q)=0$. The quasipotential wave functions (10) include projection operators on the states with definite spins: $\bar{u}_i(0)\bar{u}_j(0)=[C\hat{\varepsilon}(\gamma_5)(1+\gamma_0)]_{ij}/2\sqrt{2}$ and $v_i(0)v_j(0)=[(1-\gamma_0)\hat{\varepsilon}(\gamma_5)C]_{ij}/2\sqrt{2}$, where C is the charge conjugation matrix.

Leading order vertex functions Γ_i in (9) have the following form:

$$\Gamma^{\beta\omega}_1=\mathcal{K}_1 D_\mu{}^\beta(p_1+q_1)D_\nu{}^\omega(p_2+q_2)$$
$$\times\left(\varepsilon_1^\nu\varepsilon_2^\mu+\varepsilon_1^\mu\varepsilon_2^\nu-2g^{\mu\nu}(\varepsilon_1\varepsilon_2)\right.$$
$$-D_{\lambda\kappa}(k_1-p_1-q_1)\mathfrak{E}_1^{\lambda\mu}(p_1+q_1)\mathfrak{E}_2^{\kappa\nu}(p_2+q_2)$$
$$\left.-D_{\kappa\lambda}(k_1-p_2-q_2)\mathfrak{E}_1^{\kappa\nu}(p_2+q_2)\mathfrak{E}_2^{\lambda\mu}(p_1+q_1)\right),$$
$$\Gamma^{\beta\omega\theta}_2=\mathcal{K}_2\mathfrak{E}_2^\mu(-k_1)D_\mu{}^\beta(k_1+k_2)D^{\theta\omega}(p_1+q_1)$$
$$\times\frac{m_b-\hat{p}_1-\hat{q}_1-\hat{q}_2}{(p_1+q_1+q_2)^2-m_b^2}$$
$$+\mathcal{K}_5\varepsilon_2^\omega\mathfrak{E}_1^{\mu\nu}(p_1+q_1)D_\mu{}^\beta(k_1-p_1-q_1)D_\nu{}^\theta(p_1+q_1)$$
$$\times\frac{m_b+k_2-q_2}{(k_2-q_2)^2-m_b^2}$$
$$+D^{\theta\beta}(p_1+q_1)\frac{m_b+\hat{p}_1+\hat{p}_2+\hat{q}_1}{(p_1+p_2+q_1)^2-m_b^2}$$
$$\times\left(\mathcal{K}_2\mathfrak{E}_1^\mu(-k_2)D_\mu{}^\omega(k_1+k_2)+\mathcal{K}_9\varepsilon_1^\omega\hat{\varepsilon}_2\frac{m_b+\hat{k}_1-\hat{q}_2}{(k_1-q_2)^2-m_b^2}\right.$$
$$\left.+\mathcal{K}_7\varepsilon_2^\omega\hat{\varepsilon}_1\frac{m_b+\hat{k}_2-\hat{q}_2}{(k_2-q_2)^2-m_b^2}\right),$$
$$\Gamma^{\beta\omega}_4=\mathcal{K}_3D^{\beta\omega}(k_1-p_1-q_1)\frac{m_b+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m_b^2}\hat{\varepsilon}_2$$
$$-\mathcal{K}_4\varepsilon_2^\omega D^{\beta\mu}(k_1-p_1-q_1)\frac{m_b-\hat{k}_2+\hat{q}_2}{(k_2-q_2)^2-m_b^2}\gamma_\mu$$
$$-\mathcal{K}_5\mathfrak{E}_2^{\mu\nu}(p_2+q_2)D_\mu{}^\beta(k_1-p_1-q_1)D_\nu{}^\omega(p_2+q_2),$$
$$\Gamma^{\beta\omega}_5=\mathcal{K}_6D^{\beta\omega}(p_2+q_2)\frac{m_c+\hat{k}_2-\hat{q}_1}{(k_2-q_1)^2-m_c^2}\hat{\varepsilon}_2$$
$$+\mathcal{K}_7\varepsilon_2^\beta D_\mu{}^\omega(p_2+q_2)\frac{m_c-\hat{p}_2-\hat{q}_1-\hat{q}_2}{(p_2+q_1+q_2)^2-m_c^2}\gamma_\mu,$$
$$\Gamma^{\beta\omega}_8=\mathcal{K}_8D^{\beta\omega}(p_2+q_2)\frac{m_c+\hat{k}_1-\hat{q}_1}{(k_1-q_1)^2-m_c^2}\hat{\varepsilon}_1$$
$$+\mathcal{K}_9\varepsilon_1^\beta D_\mu{}^\omega(p_2+q_2)\frac{m_c-\hat{p}_2-\hat{q}_1-\hat{q}_2}{(p_2+q_1+q_2)^2-m_c^2}\gamma_\mu$$
$$-\mathcal{K}_{10}\mathfrak{E}_1^{\mu\nu}(p_2+q_2)D_\mu{}^\beta(k_1-p_2-q_2)D_\nu{}^\omega(p_2+q_2),$$
$$\Gamma^{\beta\omega}_{10}=\mathcal{K}_{11}D^{\beta\omega}(k_1-p_2-q_2)\frac{m_c+\hat{k}_2-\hat{p}_1}{(k_2-p_1)^2-m_c^2}\hat{\varepsilon}_2$$
$$+\mathcal{K}_3\varepsilon_2^\omega D^{\beta\mu}(k_1-p_2-q_2)\frac{m_c-\hat{k}_2+\hat{q}_1}{(k_2-q_1)^2-m_c^2}\gamma_\mu$$
$$+\mathcal{K}_{10}\mathfrak{E}_2^{\mu\nu}(p_1+q_1)D_\mu{}^\beta(k_1-p_2-q_2)D_\nu{}^\omega(p_1+q_1), \quad (11)$$

where we introduce the following tensors:

$$\mathfrak{E}^{\mu\nu}_{1,2}(x)=g^{\mu\nu}(k_{1,2}-2x)\varepsilon_{1,2}+\varepsilon^\mu_{1,2}(2k^\nu_{1,2}-x^\nu)$$
$$+\varepsilon^\nu_{1,2}(k^\mu_{1,2}+x^\mu),$$
$$\mathfrak{E}^\mu_{1,2}(x)=\varepsilon^\nu_{2,1}\mathfrak{E}^{\mu\nu}_{1,2}(x), \quad (12)$$

and $D_{\mu\nu}(k)$ is the gluon propagator, which is taken in the Feynman gauge. Other vertex functions Γ_i can be obtained by means of the simultaneous replacement $m_c\leftrightarrow m_b$, $p_1\leftrightarrow p_2$, and $q_1\leftrightarrow q_2$ in Eqs. (11):

$$\Gamma_3^{\beta\omega\theta} = \Gamma_2^{\beta\omega\theta}\Big|_{\substack{m_b \rightleftharpoons m_c,\\ p_1\rightleftharpoons p_2\\ q_1\rightleftharpoons q_2}}, \quad \Gamma_6^{\beta\omega} = \Gamma_4^{\beta\omega}\Big|_{\substack{m_b \rightleftharpoons m_c,\\ p_1\rightleftharpoons p_2\\ q_1\rightleftharpoons q_2}},$$
$$\Gamma_7^{\beta\omega} = \Gamma_5^{\beta\omega}\Big|_{\substack{m_b \rightleftharpoons m_c,\\ p_1\rightleftharpoons p_2\\ q_1\rightleftharpoons q_2}}, \tag{13}$$
$$\Gamma_9^{\beta\omega} = \Gamma_8^{\beta\omega}\Big|_{\substack{m_b \rightleftharpoons m_c,\\ p_1\rightleftharpoons p_2\\ q_1\rightleftharpoons q_2}}, \quad \Gamma_{11}^{\beta\omega} = \Gamma_{10}^{\beta\omega}\Big|_{\substack{m_b \rightleftharpoons m_c.\\ p_1\rightleftharpoons p_2\\ q_1\rightleftharpoons q_2}}$$

Color factors of the Feynman amplitudes should be contracted over color indices with antisymmetric color functions $\epsilon^{c_1c_2A}/\sqrt{2}$ and $\epsilon^{c_3c_4B}/\sqrt{2}$ $(c_i, A, B = 1,2,3)$ of D_{bc} and $\bar{D}_{\bar{b}\bar{c}}$ diquarks. As a result we obtain the 11 different color factors $\mathcal{K}_i$ in (11), which can be presented as follows:

$$\mathcal{K}_1 = -3\mathcal{C}_0 - 3\mathcal{C}_1 + 4\mathcal{C}_3, \quad \mathcal{K}_2 = \frac{4}{3}\mathcal{C}_1,$$
$$\mathcal{K}_3 = \frac{2i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - 4\mathcal{C}_2),$$
$$\mathcal{K}_4 = \frac{i}{3}(\mathcal{C}_0 - \mathcal{C}_1 - \mathcal{C}_2), \quad \mathcal{K}_5 = \frac{3}{2}\mathcal{C}_0 + \mathcal{C}_1 - 2\mathcal{C}_3,$$
$$\mathcal{K}_6 = -\frac{i}{3}(\mathcal{C}_0 + 3\mathcal{C}_1 - 5\mathcal{C}_2),$$
$$\mathcal{K}_7 = \frac{2i}{3}(\mathcal{C}_0 - 2\mathcal{C}_2), \quad \mathcal{K}_8 = -\frac{i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - 5\mathcal{C}_2),$$
$$\mathcal{K}_9 = \frac{2i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - 2\mathcal{C}_2),$$
$$\mathcal{K}_{10} = \frac{3}{2}\mathcal{C}_0 + 2\mathcal{C}_1 - 2\mathcal{C}_3,$$
$$\mathcal{K}_{11} = -\frac{i}{3}(\mathcal{C}_0 + 2\mathcal{C}_1 - \mathcal{C}_2),$$
$$\mathcal{C}_0 = \delta^{g_1g_2}\delta_{AB}, \quad \mathcal{C}_1 = if^{g_1g_2a}(T^a)_{BA},$$
$$\mathcal{C}_2 = (T^{g_1}T^{g_2})_{BA},$$
$$\mathcal{C}_3 = f^{g_1ea}f^{g_2eb}(T^aT^b)_{BA}, \tag{14}$$

where $g_{1,2} = 1, \ldots, 8$ are the color indices of the initial gluons, A and B are the color indices of the final diquarks.

Let us present here, for example, the transformation of the first amplitude in Fig. 1 from $\mathcal{T}_1(p_1, p_2; q_1, q_2)$ in (2) to $\mathcal{M}_1(k_1, k_2; P, Q)$ in (8) which takes the form in the Feynman gauge

$$\mathcal{T}_1(p_1, p_2; q_1, q_2) = -8i\pi^2\alpha_s^2 f^{g_1g_2b}(T^a)_{c_1c_3}$$
$$\times (T^b)_{c_2c}(T^a)_{cc_4}\epsilon^{c_1c_2A}\epsilon^{c_3c_4B}\left[\bar{u}(p_1)\gamma_\alpha v(q_1)\right]$$
$$\times \left[\bar{u}(p_2)\gamma_\beta\frac{m_b - \hat{p}_1 - \hat{q}_1 - \hat{q}_2}{(p_1+q_1+q_2) - m_b^2}\gamma_\omega v(q_2)\right]$$
$$\times \varepsilon_1^\mu(k_1)\varepsilon_2^\nu(k_2)D^{\alpha\omega}(p_1+q_1)D^{\rho\beta}(k_1+k_2)$$
$$\times (g_{\mu\nu}(k_2-k_1)_\rho - g_{\nu\rho}(k_1+2k_2)_\mu + g_{\mu\rho}(2k_1+k_2)_\nu), \tag{15}$$

$$\mathcal{M}_1(k_1, k_2; P, Q) = -\frac{4}{3}i\pi^2\alpha_s^2\sqrt{M_{D_{bc}}M_{\bar{D}_{\bar{b}\bar{c}}}}\, f^{g_1g_2a}(T^a)_{BA}$$
$$\times \int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{\bar{\Psi}^0_{D_{bc}}(\mathbf{p})}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{(\epsilon_c(p)+m_c)}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{(\epsilon_b(p)+m_b)}{2m_b}}}$$
$$\times\frac{\bar{\Psi}^0_{\bar{D}_{\bar{b}\bar{c}}}(\mathbf{q})}{\sqrt{\frac{\epsilon_c(q)}{m_c}\frac{(\epsilon_c(q)+m_c)}{2m_c}\frac{\epsilon_b(q)}{m_b}\frac{(\epsilon_b(q)+m_b)}{2m_b}}}$$
$$\times\mathrm{Tr}\left\{\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_c(\epsilon_c(p)+m_c)}+\frac{\hat{p}}{2m_c}\right]\Sigma^P(1+\hat{v}_1)\right.$$
$$\times\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m_b(\epsilon_b(p)+m_b)}-\frac{\hat{p}}{2m_b}\right]$$
$$\times\gamma_\beta\frac{m_b-\hat{p}_1-\hat{q}_1-\hat{q}_2}{(p_1+q_1+q_2)-m_b^2}\gamma_\omega\left[\frac{\hat{v}_2-1}{2}\right.$$
$$\left.+\hat{v}_2\frac{\mathbf{q}^2}{2m_b(\epsilon_b(q)+m_b)}-\frac{\hat{q}}{2m_b}\right]\Sigma^Q(1+\hat{v}_2)$$
$$\left.\times\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m_c(\epsilon_c(q)+m_c)}+\frac{\hat{q}}{2m_c}\right]\gamma_\alpha\right\}$$
$$\times\varepsilon_1^\mu(k_1)\varepsilon_2^\nu(k_2)D^{\alpha\omega}(p_1+q_1)D^{\rho\beta}(k_1+k_2)$$
$$\times\left(g_{\mu\nu}(k_2-k_1)_\rho - g_{\nu\rho}(k_1+2k_2)_\mu + g_{\mu\rho}(2k_1+k_2)_\nu\right). \tag{16}$$

The production amplitude (9) and vertex functions (11) contain relative momenta p and q in exact form. In order to take into account relativistic corrections of second order in p and q we expand all inverse denominators of the quark and gluon propagators as follows:

$$\frac{1}{(p_{1,2}+q_{1,2})^2}$$
$$= \frac{1}{s\,\eta_{1,2}^2}\left[1 \mp \frac{2(pQ+qP)}{s\,\eta_{1,2}} - \frac{p^2+2pq+q^2}{s\,\eta_{1,2}^2} + \cdots\right],$$
$$\times\frac{1}{(p_1+q_1+q_2)^2 - m_b^2}$$
$$= \frac{1}{Z_1}\left[1 - \frac{2pQ+p^2}{Z_1} + \frac{4(pQ)^2}{Z_1^2} + \cdots\right],$$
$$\times\frac{1}{(k_2-q_1)^2 - m_c^2}$$
$$= \frac{1}{Z_2}\left[1 + \frac{2k_2q-q^2}{Z_2} + \frac{4(k_2Q)^2}{Z_2^2} + \cdots\right], \tag{17}$$

where $Z_1 = s\,\eta_1 + \eta_2^2M^2 - m_b^2$ and $Z_2 = t\,\eta_1 - \eta_1\eta_2M^2 - m_c^2$. The amplitude (9) contains 16 different denominators to be expanded in the manner of Eq. (17). Neglecting the bound state corrections, we find that an expansion of denominators takes one of the following forms: $s\,\eta_{1,2}$, $s\,\eta_{1,2}^2$, $\eta_{1,2}(M^2-t)$ or $\eta_{1,2}(M^2-s-t)$. Then, taking into account kinematical restrictions on s and t,

$$4M^2 \le s, \quad \left|t + \frac{s}{2} - M^2\right| \le \frac{s}{2}\sqrt{1 - \frac{4M^2}{s}}, \tag{18}$$

and nonrelativistic estimate $\eta_1 \approx m_c/(m_c+m_b) \approx 1/4$ for (bc) diquarks, we conclude that the expansion parameters in (17) are at least as small as $4p^2/M^2$ and $4q^2/M^2$.

Preserving in the expanded amplitude terms up to second order both in relative momenta p and q, we can perform the angular integration using the following relations for $\mathcal{S}$-wave diquarks:

$$\int \frac{\Psi_0^{\mathcal{S}}(\mathbf{p})}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}\frac{d\mathbf{p}}{(2\pi)^3} = \frac{1}{\sqrt{2}\pi}\int_0^\infty \frac{p^2 R_{\mathcal{S}}(p)}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}dp,$$

$$\int \frac{p_\mu p_\nu \Psi_0^{\mathcal{S}}(\mathbf{p})}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}\frac{d\mathbf{p}}{(2\pi)^3} = -\frac{g_{\mu\nu}-v_{1\mu}v_{1\nu}}{3\sqrt{2}\pi}\int_0^\infty \frac{p^4 R_{\mathcal{S}}(p)}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}dp, \tag{19}$$

where $R_{\mathcal{S}}(p)$ is the radial wave function.

In order to calculate the cross section we have to sum the squared modulus of the amplitude over final particle polarizations in the case of pair axial-vector diquark production and average it over polarizations of the initial gluons using the following relations:

$$\sum_\lambda \varepsilon_P^\mu \varepsilon_P^{*\,\nu} = v_1^\mu v_1^\nu - g^{\mu\nu},$$
$$\sum_\lambda \varepsilon_Q^\mu \varepsilon_Q^{*\,\nu} = v_2^\mu v_2^\nu - g^{\mu\nu}, \tag{20}$$
$$\sum_\lambda \varepsilon_{1,2}^\mu \varepsilon_{1,2}^{*\,\nu} = \frac{k_1^\mu k_2^\nu + k_1^\nu k_2^\mu}{k_1\cdot k_2} - g^{\mu\nu}.$$

Then we also average it over the colors of the initial gluons and sum over the diquark color indices A and B. Finally, we obtain the following expression for the differential cross section of double heavy diquark pair production:

$$d\sigma[gg\to D_{bc}+\bar{D}_{\bar{b}\bar{c}}](s,t) = \frac{\pi M^2\alpha_s^4}{65{,}536\, s^2}|\tilde{R}(0)|^4 \times\Big[F^{(1)}(s,t) - 4(\omega_{01}+\omega_{10}-\omega_{11})F^{(1)}(s,t) - 4m_c^{-1}m_b^{-1}(m_c^2\omega_{\frac12\frac32}+m_b^2\omega_{\frac32\frac12})F^{(1)}(s,t) + 6(\omega_{01}+\omega_{10})^2F^{(1)}(s,t) + \omega_{\frac12\frac12} \times(1-3\omega_{01}-3\omega_{10})F^{(2)}(s,t) + \omega_{\frac12\frac12}^2F^{(3)}(s,t)\Big], \tag{21}$$

where the parameter $\tilde{R}(0)$ in (21) has the following form:

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}}\int_0^\infty \sqrt{\frac{(\epsilon_c(p)+m_c)(\epsilon_b(p)+m_b)}{2\epsilon_c(p)\,2\epsilon_b(p)}}R(p)p^2\,dp. \tag{22}$$

It represents the relativistic generalization of the value of wave function at the origin $R(0)$. The relativistic parameters ω_{nk} are expressed through momentum integrals with the double heavy diquark radial wave function $R(p)$:

$$I_{nk} = \int_0^{m_c} p^2R(p)\sqrt{\frac{(\epsilon_c(p)+m_c)(\epsilon_b(p)+m_b)}{2\epsilon_c(p)\,2\epsilon_b(p)}} \left(\frac{\epsilon_c(p)-m_c}{\epsilon_c(p)+m_c}\right)^n\left(\frac{\epsilon_b(p)-m_b}{\epsilon_b(p)+m_b}\right)^k dp,$$
$$\omega_{nk} = \sqrt{\frac{2}{\pi}}\frac{I_{nk}}{\tilde{R}(0)}. \tag{23}$$

In contrast to our previous work [40] there are terms in (21) which contain relativistic parameters ω_{nk} with fractional indices. They appear if we preserve the symmetry of the cross section (21) in the quark masses m_c and m_b. The auxiliary functions $F^{(i)}(s,t)$ contain a nonrelativistic contribution and relativistic corrections to the cross section connected with the relative motion of heavy quarks. Their exact analytical expressions are extremely lengthy in the case of diquarks of different flavors b and c, so we present them in Appendix A only in the case of (cc) diquarks.

3 Numerical results and discussion

The quasipotential wave functions of double heavy diquarks are obtained by numerical solution of the Schrödinger equation with effective relativistic Hamiltonian based on the QCD generalization of the Breit potential completed by scalar and vector exchange confinement terms, as is described in detail in our previous work [27–29,40]. We present the values of the diquark masses and relativistic parameters (22), (23) in Table 1. Numerical masses of charmonium and B_c mesons obtained in our model are in good agreement with existing experimental data (the difference is less than 1 %) [14–20,27–29,40]. Analogously, the masses for the (bc) and (cc) double heavy diquarks from Table 1 coincide with the estimates made in other approaches [33–38,56–59]. Note that our definition (23) of relativistic integrals I_{nk} contains a cut-off at the value of the c-quark mass $\Lambda = m_c$. Although the integrals (23) are convergent, there are some uncertainties in their calculation related with the determination of the wave function in the region of relativistic momenta $p \gtrsim m_c$ in our model.

The numerical results for the total cross section of double heavy diquark pair production corresponding to the LHC relative energies $\sqrt{S} = 7$ and 14 TeV are presented in Table 2. The integration in (1) is performed with partonic distribution functions from CTEQ5L and CTEQ6L1 sets [60,61]. The renormalization and factorization scales are set equal to the transverse mass $\mu = m_T = \sqrt{M^2+P_T^2}$. The leading

Table 1 Numerical values of parameters describing double heavy (cc) and (bc) diquarks

Diquark state	$n^{2S+1}L_J$	M, GeV	$\tilde{R}(0)$, GeV$^{3/2}$	ω_{10}	ω_{01}	$\omega_{\frac{1}{2}\frac{1}{2}}$	ω_{11}	$\omega_{\frac{1}{2}\frac{3}{2}}$	$\omega_{\frac{3}{2}\frac{1}{2}}$
SD_{bc}	1^1S_0	6.517	0.50	0.0383	0.0045	0.0131	0.00039	0.00014	0.0011
AVD_{bc}	1^3S_1	6.526	0.48	0.0384	0.0045	0.0132	0.00038	0.00013	0.0011
AVD_{cc}	1^3S_1	3.224	0.38	0.0323			0.0023		

Table 2 The value of the cross sections of double heavy diquark pair production

Energy $\sqrt{S}$	Diquark pair	CTEQ5L		CTEQ6L1	
		σ_{nonrel}, nb	σ_{rel}, nb	σ_{nonrel}, nb	σ_{rel}, nb
$\sqrt{S}=7$ TeV	$SD_{bc}+S\bar{D}_{\bar{b}\bar{c}}$	0.063	0.018	0.057	0.016
	$AVD_{bc}+AV\bar{D}_{\bar{b}\bar{c}}$	0.25	0.053	0.23	0.049
	$AVD_{cc}+AV\bar{D}_{\bar{c}\bar{c}}$	1.39	0.28	1.07	0.22
$\sqrt{S}=14$ TeV	$SD_{bc}+S\bar{D}_{\bar{b}\bar{c}}$	0.14	0.039	0.12	0.034
	$AVD_{bc}+AV\bar{D}_{\bar{b}\bar{c}}$	0.55	0.12	0.48	0.10
	$AVD_{cc}+AV\bar{D}_{\bar{c}\bar{c}}$	2.51	0.51	1.94	0.40

Table 3 Double heavy diquark production cross sections corresponding to the rapidity range $2 < y_{P,Q} < 4.5$

Energy $\sqrt{S}$	Diquark pair	CTEQ5L		CTEQ6L1	
		σ_{nonrel}, nb	σ_{rel}, nb	σ_{nonrel}, nb	σ_{rel}, nb
$\sqrt{S}=7$ TeV	$SD_{bc}+S\bar{D}_{\bar{b}\bar{c}}$	0.010	0.003	0.009	0.003
	$AVD_{bc}+AV\bar{D}_{\bar{b}\bar{c}}$	0.032	0.007	0.029	0.006
	$AVD_{cc}+AV\bar{D}_{\bar{c}\bar{c}}$	0.19	0.038	0.14	0.029
$\sqrt{S}=14$ TeV	$SD_{bc}+S\bar{D}_{\bar{b}\bar{c}}$	0.024	0.007	0.020	0.006
	$AVD_{bc}+AV\bar{D}_{\bar{b}\bar{c}}$	0.076	0.016	0.066	0.014
	$AVD_{cc}+AV\bar{D}_{\bar{c}\bar{c}}$	0.35	0.072	0.25	0.053

order result for the strong coupling constant $\alpha_s(\mu)$ with initial value $\alpha_s(\mu=M_Z)=0.118$ is used. In a second stage the diquark nucleus can join with high probability a light quark and form a double heavy baryon. In the nonrelativistic limit all parameters ω_{nk} are equal to zero and only the $F^{(1)}(s,t)$ term survives in square brackets of (21). Then, replacing $\tilde{R}(0)$ by a nonrelativistic value of the radial wave function at the origin $R(0)=\sqrt{2/\pi}\int p^2R(p)\,\mathrm{d}p$ and assuming that the diquark mass is equal to the sum of masses of the constituent quarks $M_0=m_b+m_c$, we obtain our nonrelativistic prediction for the double heavy diquark pair production cross section presented in the third and fifth columns of Tables 2 and 3. In our model we obtain the following nonrelativistic values: $R(0)=0.67$ GeV$^{3/2}$ and $R(0)=0.53$ GeV$^{3/2}$ for the (bc) and (cc) diquarks, respectively, which lie close to the results $R(0)=0.73$ GeV$^{3/2}$ and $R(0)=0.53$ GeV$^{3/2}$ from [33–38,56–58]. In order to obtain the cross sections for the axial-vector (cc) diquark pair production we replace $m_b\to m_c$ in all expressions and multiply the amplitude by an additional factor $1/4$ ($1/16$ in the cross section) according to the Pauli exclusion principle.

As follows from the results presented in Tables 2 and 3, relativistic effects by almost five times decrease the values of the (bc) and (cc) double heavy diquark pair production cross sections. The main role in such a decrease plays the difference between the relativistic parameter, $\tilde{R}(0)$, and the nonrelativistic one, $R(0)$. $\tilde{R}(0)$ ($R(0)$) enters the corresponding cross section in fourth degree, so that even a small modification of this parameter caused by relativistic corrections in the Breit potential leads to a substantial change in the cross section. For example, in the case of the axial-vector (bc) diquark $\tilde{R}(0)$ is only 25 % smaller than its nonrelativistic value, but this difference results in a more than three times decrease of the value of the cross section. The bound state effects connected with the non-zero diquark bound state energy $W=M-m_c-m_b\neq 0$ bring about an additional 30 % decrease. Finally, the relativistic corrections originating from the expansion of the production amplitude increase the value of the cross section by 10–20 %, which is insufficient to compensate the large negative contributions from the first two sources.

In Fig. 3 we present the results of our calculation of the differential cross section in terms of the rapidity $y_P=\frac{1}{2}\ln\frac{P_0+P_\parallel}{P_0-P_\parallel}$. The rapidities of the outcoming diquarks with momenta P and Q can be obtained in the form

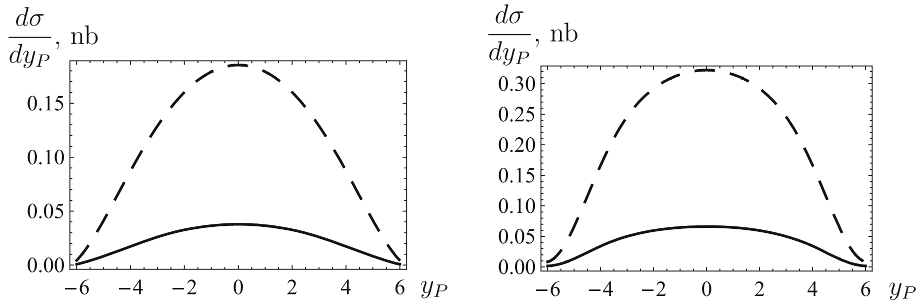

Fig. 3 The differential cross sections for $pp \to D\bar{D} + X$ at $\sqrt{S} = 7$ TeV (*left*) and $\sqrt{S} = 14$ TeV (*right*) as functions of rapidity y_P. *Solid* and *dashed curves* represent total and nonrelativistic results, respectively

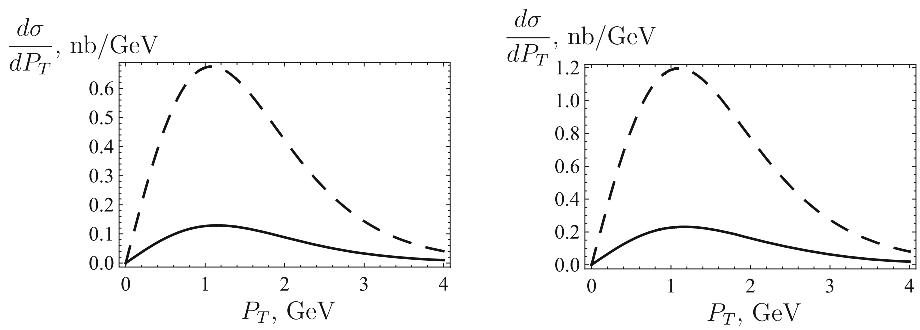

Fig. 4 The differential cross sections for $pp \to D\bar{D} + X$ at $\sqrt{S} = 7$ TeV (*left*) and $\sqrt{S} = 14$ TeV (*right*) as functions of transverse momentum P_T of diquark pair integrated over the rapidity. *Solid* and *dashed curves* represent total and nonrelativistic results, respectively

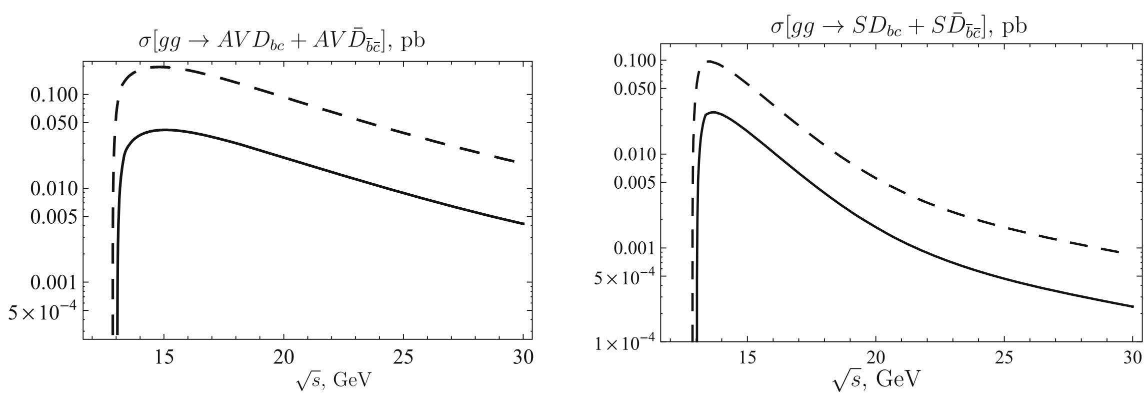

Fig. 5 Cross section of diquark–antidiquark gluonic production as a function of their invariant mass. *Solid* and *dashed curves* represent total and nonrelativistic results, respectively

$$y_{P,Q} = \frac{1}{2}\ln\frac{x_1}{x_2} \pm \frac{1}{2}\ln\left[\frac{s}{M^2 - t} - 1\right]. \tag{24}$$

The differential cross section $d\sigma/dy_P$ shown in Fig. 3 can be important for a comparison with forthcoming experimental data. It is clear from this plot that relativistic effects strongly influence the rapidity distribution of the final diquarks. In the LHCb experiment [21] the rapidity lies in the range $2 < y_{P,Q} < 4.5$, so we should integrate the differential cross section (1) over the rapidities from such an interval in order to obtain the value corresponding to the experiment at the LHCb detector. These results are presented in Table 3. We show in Fig. 4 the distribution over the transverse momentum of the diquarks integrated over all rapidities at $\sqrt{S} = 7$ TeV. It can be seen in Fig. 4 that the account of relativistic corrections leads to the ratio of relativistic and nonrelativistic cross sections $\sigma_{\rm rel}/\sigma_{\rm nr} \approx 0.2$ near the peak. This trend remains unchanged in the region of high transverse momenta. In order to have a more complete concept about production processes we show in Fig. 5 the cross section of double diquark production in the gluonic subprocess as a function of its invariant mass. As follows from Figs. 4 and 5, a typical p_T momentum is of order 1.2 GeV and total typical momenta of diquarks and antidiquarks are more than 2.5 GeV. The most part of the pair (bc) diquark production cross section is accumulated in that region of $\sqrt{s}$ which corresponds to large momenta $|\mathbf{P}| \geq 2.5$ GeV: 70 % for scalar diquark pairs and 85 % for axial-vector diquark pairs. But the probability $|\Psi_0^S(\mathbf{p})|^2$ to find quarks with relativistic relative momentum $p \geq 1.5$ GeV is strongly suppressed (we use a cutoff for the momentum integrals in (23) at $m_c = 1.55$ GeV). This follows from the obtained relativistic wave functions in our model which have maximum values at $p, q \sim 0.4$ GeV. So, we could expect that four heavy quarks and antiquarks are not sufficiently close in the phase space and rescattering effects between heavy quarks and antiquarks are not large, but they should be investigated additionally.

Let us estimate the total theoretical uncertainty of the obtained results. The first and main source of the uncertainty is connected with the relativistic parameter $\tilde{R}(0)$, which enters the cross section in fourth degree and defines the order of magnitude of the final result. The accuracy of this parameter depends directly on the error in the determination of the relativistic quasipotential wave function in our model, which we estimate to be 10 %. Of course, this estimate is a very approximate one but it can be justified by the better than 1 % accuracy of the calculation of charmonium mass spectrum. Then we estimate the error in the cross section from this source to be not exceeding 40 %. The next source of uncertainty deals with the corrections of fourth and highest order, which are truncated in our amplitude expansions (17). As mentioned before, the corrections of second order give a 10–20 % contribution to the value of the cross section, so we suppose that 20 % will be a reasonable estimate for this error. The contribution of the next-to-leading order in the strong coupling constant α_s is difficult to estimate. It depends significantly on the structure of the Feynman amplitude and has to be calculated independently. For example, it is well known that such corrections lead to a significantly increasing factor $K = 1.6 \div 1.9$ to the cross section of charmonium pair production in e^+e^- annihilation [9–11,62]. On the other hand, recently it was found that the NLO α_s contribution to the cross section of J/ψ pair production in pp-collisions for the LHCb rapidity range amounts to a value of order 10 % [63]. So, we assume that a similar contribution occurs in diquark pair production in pp-collisions. Finally, there is one additional uncertainty connected with the accuracy of partonic distribution functions, which was estimated to be 15 % in [27–29]. Then, adding all the mentioned uncertainties in quadrature, we obtain the total error in 48 % for our results.

In the case of single quarkonium (or diquark) production the color octet (or color sextuplet) contribution is comparable with the contribution of color triplet (or antitriplet) state [41–43,64–66]. At the same time, as we mentioned above, for pair charmonium production the color octet mechanism gives an essentially smaller value to the cross section at small or intermediate momenta p_T [26]. Despite the fact that the account of color octet states in the single J/ψ production in the pp-interaction can explain the value of the measured cross section, it predicts a substantial transverse component for the polarization of J/ψ mesons. This is in disagreement with the CDF J/ψ polarization measurement. More recent theoretical studies have considered the addition of the $gg \to J/\psi c\bar{c}$ process in CSM or higher order corrections in α_s: $gg \to J/\psi gg$, $gg \to J/\psi ggg$. With these additional processes the discrepancy between the theoretical prediction and experimental measurements significantly decreases. So, in our opinion, the account of color-sextuplet states in inclusive double heavy diquark pair production should be done together with a consideration of order α_s corrections. In this work we omit terms of order $O(\alpha_s)$. Our estimate of this correction of order $O(\alpha_s)$ includes also the color-sextuplet contribution.

Acknowledgments The authors are grateful to D. Ebert, R.N. Faustov, and V.O. Galkin for useful discussions. The work is supported partially by the Ministry of Education and Science of Russian Federation under Competitiveness Enhancement Program 2013-2020 and Dynasty Foundation.

Appendix A: The coefficients $F^{(i)}$ entering the differential cross section (21) for pair axial-vector diquark (cc) production

Exact analytical expressions for the functions $F^{(i)}(s,t)$ in (21) are extremely lengthy for heavy quarks of different flavor, so we present here only their analytical expressions in the case of pair axial-vector diquark (cc) production. In these expressions we take into account linear effects in the bound state energy W of heavy quarks and introduce the notation $M_c = 2m_c$. Bound state effects are taken into account in numerical results from Tables 2 and 3. We have

$$F^{(i)}(s,t) = F_0^{(i)}(s,t) + W\, F_1^{(i)}(s,t), \tag{25}$$

$$\begin{aligned}
&F_0^{(1)}[AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}](s,t)\\
&\quad = \frac{524288}{27 M_c^4 s^6 \left(M_c^2 - t\right)^4 \left(M_c^2 - s - t\right)^4}\\
&\quad \times \Big[27648 M_c^{24} - 72 M_c^{22}(1595 s + 4596 t)\\
&\quad + 3 M_c^{20}(67687 s^2 + 437088 st + 605232 t^2)\\
&\quad - 8 M_c^{18}(28007 s^3 + 278328 s^2 t + 849501 st^2 + 754920 t^3)\\
&\quad + 4 M_c^{16}(48546 s^4 + 575480 s^3 t\\
&\quad + 2731629 s^2 t^2 + 5276664 st^3 + 3390660 t^4)\\
&\quad - 2 M_c^{14}(66854 s^5 + 867710 s^4 t + 5237453 s^3 t^2\\
&\quad + 15810492 s^2 t^3 + 21825720 st^4 + 10831968 t^5)\\
&\quad + M_c^{12}(64025 s^6 + 980113 s^5 t + 6934011 s^4 t^2\\
&\quad + 27679700 s^3 t^3 + 59798910 s^2 t^4 + 63129024 st^5\\
&\quad + 25238304 t^6) - 2 M_c^{10}(9796 s^7\\
&\quad + 190998 s^6 t + 1629993 s^5 t^2 + 8003124 s^4 t^3\\
&\quad + 23392115 s^3 t^4 + 38627220 s^2 t^5 + 32576040 st^6
\end{aligned}$$

$$+ 10803456t^7) + t^2(s+t)^2(8s^8 + 25s^7t + 2536s^6t^2$$
$$+ 21366s^5t^3 + 78759s^4t^4 + 157896s^3t^5$$
$$+ 179640s^2t^6 + 108864st^7 + 27216t^8)$$
$$- 2M_c^2t(s+t)^2(16s^8 + 243s^7t + 5526s^6t^2$$
$$+ 49040s^5t^3 + 215626s^4t^4 + 530597s^3t^5$$
$$+ 741924s^2t^6 + 546660st^7 + 163296t^8) + M_c^8$$
$$\times (4006s^8 + 94606s^7t + 1029199s^6t^2$$
$$+ 6247798s^5t^3 + 23171033s^4t^4 + 52444016s^3t^5$$
$$+ 69078684s^2t^6 + 47988288st^7 + 13491360t^8)$$
$$- 2M_c^6(322s^9 + 7064s^8t + 99306s^7t^2$$
$$+ 779460s^6t^3 + 3657884s^5t^4 + 10718238s^4t^5$$
$$+ 19496435s^3t^6 + 21114948s^2t^7$$
$$+ 12361788st^8 + 2995920t^9)$$
$$+ M_c^4(68s^{10} + 1153s^9t + 19692s^8t^2 + 217805s^7t^3$$
$$+ 1362129s^6t^4 + 5166549s^5t^5$$
$$+ 12342213s^4t^6 + 18546596s^3t^7$$
$$+16897269s^2t^8 + 8485920st^9 + 1796688t^{10})\Big], \quad (26)$$

$$F_0^{(2)}[AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}](s,t)$$
$$= -\frac{524288}{81M_c^4s^8\left(M_c^2-t\right)^5\left(M_c^2-s-t\right)^5}$$
$$\times \Big[1741824M_c^{32} - 1728M_c^{30}(5897s + 16128t)$$
$$+ 144M_c^{28}(189641s^2 + 1087056st + 1451520t^2)$$
$$- 144M_c^{26}(322301s^3 + 2802300s^2t$$
$$+ 7789176st^2 + 6773760t^3) + M_c^{24}(57554074s^4$$
$$+ 656045232s^3t + 2763480672s^2t^2 + 4972983552st^3$$
$$+ 3170119680t^4) - 2M_c^{22}(27433899s^5$$
$$+ 384167638s^4t + 2133779508s^3t^2 + 5813307936s^2t^3$$
$$+ 7628766624st^4 + 3804143616t^5)$$
$$+ 6M_c^{20}(6821962s^6 + 113839005s^5t + 781665683s^4t^2$$
$$+ 2819646560s^3t^3 + 5596217064s^2t^4$$
$$+ 5718898944st^5 + 2324754432t^6)$$
$$- 2M_c^{18}(12207465s^7 + 234642355s^6t$$
$$+ 1930593059s^5t^2$$
$$+ 8652159088s^4t^3 + 22804443900s^3t^4$$
$$+ 35210657664s^2t^5$$
$$+ 29218299264st^6 + 9963233280t^7)$$
$$+ M_c^{16}(11661347s^8 + 251850550s^7t$$
$$+ 2421795360s^6t^2 + 13072181288s^5t^3$$
$$+ 42975298740s^4t^4$$
$$+ 88311607776s^3t^5 + 110619734400s^2t^6$$
$$+ 76740162048st^7 + 22417274880t^8)$$
$$- M_c^{14}(4223561s^9 + 104555317s^8t$$
$$+ 1160158397s^7t^2 + 7399876560s^6t^3$$
$$+ 29431644460s^5t^4$$
$$+ 75678177864s^4t^5 + 126369906768s^3t^6$$
$$+ 132222824064s^2t^7 + 78350462784st^8$$
$$+ 19926466560t^9)$$
$$+ 4t^3(s+t)^3(88s^{10} + 291s^9t + 30322s^8t^2$$
$$+ 297142s^7t^3 + 1336999s^6t^4 + 3529608s^5t^5$$
$$+ 5988728s^4t^6 + 6737472s^3t^7 + 4950288s^2t^8$$
$$+ 2177280st^9 + 435456t^{10}) + M_c^{12}(1077011s^{10}$$
$$+ 31978294s^9t + 416887472s^8t^2 + 3125057808s^7t^3$$
$$+ 14810788148s^6t^4 + 46251513620s^5t^5$$
$$+ 96893779668s^4t^6 + 135324576576s^3t^7$$
$$+ 120857286384s^2t^8 + 62193643776st^9$$
$$+ 13948526592t^{10})$$
$$- M_c^2t^2(s+t)^2(1504s^{11} + 23728s^{10}t$$
$$+ 685939s^9t^2 + 7528052s^8t^3 + 42573677s^7t^4$$
$$+ 146340250s^6t^5 + 329945926s^5t^6 + 504952416s^4t^7$$
$$+ 526855224s^3t^8 + 362855808s^2t^9$$
$$+ 149506560st^{10} + 27869184t^{11}) - M_c^{10}$$
$$\times (189099s^{11} + 6711431s^{10}t + 106896474s^9t^2$$
$$+ 958886210s^8t^3 + 5412164759s^7t^4$$
$$+ 20272621428s^6t^5$$
$$+ 51752902204s^5t^6 + 90884023920s^4t^7$$
$$+ 108459347328s^3t^8 + 84070738368s^2t^9$$
$$+ 38069792640st^{10} + 7608287232t^{11}) + M_c^8(23696s^{12}$$
$$+ 902460s^{11}t + 18000829s^{10}t^2$$
$$+ 201542412s^9t^3 + 1383434183s^8t^4 + 6237258490s^7t^5$$
$$+ 19205490660s^6t^6 + 41225350568s^5t^7$$
$$+ 61985442066s^4t^8 + 64263674160s^3t^9$$
$$+ 43813561824s^2t^{10}$$
$$+ 17647352064st^{11} + 3170119680t^{12})$$
$$+ 2M_c^4t(1648s^{13} + 36454s^{12}t$$
$$+ 814068s^{11}t^2 + 10491584s^{10}t^3$$
$$+ 77792679s^9t^4 + 366446361s^8t^5$$
$$+ 1170001946s^7t^6 + 2627856120s^6t^7$$
$$+ 4228552375s^5t^8$$

$$
\begin{aligned}
&+ 4881077725s^4t^9 + 3965125248s^3t^{10}\\
&+ 2157018696s^2t^{11} + 705148416st^{12}\\
&+ 104509440t^{13})\\
&- M_c^6(1664s^{13} + 72088s^{12}t\\
&+ 1733298s^{11}t^2 + 25982195s^{10}t^3 + 228949037s^9t^4\\
&+ 1276267953s^8t^5 + 4775775315s^7t^6\\
&+ 12428268880s^6t^7 + 22908511678s^5t^8\\
&+ 29980767340s^4t^9\\
&+ 27364070472s^3t^{10} + 16589102400s^2t^{11}\\
&+5996839104st^{12} + 975421440t^{13})\Big], \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
&F_0^{(3)}[AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}](s,t)\\
&= \frac{262144}{243M_c^4s^{10}\left(M_c^2-t\right)^6\left(M_c^2-s-t\right)^6}\\
&\times\Big[222953472M_c^{40} - 27869184M_c^{38}(59s + 160t)\\
&+ 3456M_c^{36}(1634233s^2 + 9209088st + 12257280t^2)\\
&- 576M_c^{34}(21154723s^3+183547236s^2t+506435328st^2\\
&+ 441262080t^3)\\
&+ 48M_c^{32}(390910561s^4\\
&+ 4582950624s^3t + 19453613688s^2t^2\\
&+ 35059433472st^3 + 22504366080t^4)\\
&- 4M_c^{30}(5537521813s^5\\
&+ 81274350792s^4t + 466630287984s^3t^2\\
&+ 1292676789888s^2t^3 + 1712700702720st^4\\
&+ 864167657472t^5) + 4M_c^{28}(5240554941s^6\\
&+ 91437358522s^5t + 659112431964s^4t^2\\
&+ 2471543238528s^3t^3 + 5029243275744s^2t^4\\
&+ 5227645796352st^5 + 2160419143680t^6)\\
&- M_c^{26}(16330544031s^7 + 326792003628s^6t\\
&+ 2815199542892s^5t^2 + 13284503935680s^4t^3\\
&+ 36610822146240s^3t^4 + 58396559171328s^2t^5\\
&+ 49627101855744st^6 + 17283353149440t^7)\\
&+ M_c^{24}(10537886063s^8 + 237559033812s^7t\\
&+ 2365887361944s^6t^2 + 13397155539088s^5t^3\\
&+ 46551035169840s^4t^4 + 100601034937344s^3t^5\\
&+ 131082206097024s^2t^6 + 93716708327424st^7\\
&+ 28085448867840t^8) - 2M_c^{22}(2784626670s^9\\
&+ 70737215246s^8t + 800148664831s^7t^2\\
&+ 5268464876020s^6t^3 + 22039344056906s^5t^4\\
&+ 60145366265904s^4t^5 + 106205291855712s^3t^6\\
&+ 116324448634368s^2t^7 + 71451599192064st^8\\
&+ 18723632578560t^9) + M_c^{20}(2378426106s^{10}\\
&+ 68538265551s^9t + 877306323899s^8t^2\\
&+ 6598930104580s^7t^3 + 32238538637660s^6t^4\\
&+ 106201836358040s^5t^5 + 237119424205104s^4t^6\\
&+ 352010470800384s^3t^7 + 330977605825152s^2t^8\\
&+ 177504964116480st^9 + 41191991672832t^{10})\\
&- M_c^{18}(815322191s^{11} + 26692617042s^{10}t\\
&+ 387726963435s^9t^2 + 3311677836814s^8t^3\\
&+ 18562148953601s^7t^4 + 71659299951396s^6t^5\\
&+ 193566257649996s^5t^6 + 363728035723392s^4t^7\\
&+ 463561912782144s^3t^8 + 380261571985152s^2t^9\\
&+ 180350463541248st^{10} + 37447265157120t^{11})\\
&+ 4t^4(s+t)^4(2904s^{12} + 10131s^{11}t + 1088812s^{10}t^2\\
&+ 12565370s^9t^3 + 71302945s^8t^4 + 257354136s^7t^5\\
&+ 650793576s^6t^6 + 1191722688s^5t^7\\
&+ 1576367280s^4t^8 + 1465413120s^3t^9\\
&+ 906204672s^2t^{10}\\
&+ 334430208st^{11} + 55738368t^{12})\\
&+ M_c^{16}(220505891s^{12} + 8239519834s^{11}t\\
&+ 136412070144s^{10}t^2 + 1324461974452s^9t^3\\
&+ 8455632770450s^8t^4 + 37581776673544s^7t^5\\
&+ 119310422197680s^6t^6 + 271757302270944s^5t^7\\
&+ 438797604649680s^4t^8 + 487891081751040s^3t^9\\
&+ 353665494179712s^2t^{10} + 149887606063104st^{11}\\
&+ 28085448867840t^{12}) - M_c^2t^3(s+t)^3(52864s^{13}\\
&+ 808320s^{12}t + 29421610s^{11}t^2 + 375769223s^{10}t^3\\
&+ 2557032615s^9t^4 + 11233867505s^8t^5\\
&+ 34796810259s^7t^6 + 79006480208s^6t^7\\
&+ 132647858276s^5t^8\\
&+ 162616078080s^4t^9 + 141063190272s^3t^{10}\\
&+ 81801031680s^2t^{11}\\
&+ 28398698496st^{12} + 4459069440t^{13})\\
&- 2M_c^{14}(22481965s^{13} + 978678070s^{12}t\\
&+ 18711980507s^{11}t^2 + 207897944100s^{10}t^3\\
&+ 1512657963970s^9t^4 + 7680203547784s^8t^5\\
&+ 28156966746706s^7t^6 + 75568224170616s^6t^7\\
&+ 148153722926478s^5t^8 + 208861152514800s^4t^9\\
&+ 205204061269152s^3t^{10} + 132824617585920s^2t^{11}
\end{aligned}
$$

$$
\begin{aligned}
&+ 50738580652032st^{12} + 8641676574720t^{13})\\
&+ M_c^4 t^2 (s+t)^2 (122528s^{14} + 2904080s^{13}t\\
&+ 82920640s^{12}t^2 + 1181123451s^{11}t^3\\
&+ 9594060453s^{10}t^4 + 51055788685s^9t^5\\
&+ 192342235019s^8t^6\\
&+ 534248481092s^7t^7 + 1111820399652s^6t^8\\
&+ 1729605262024s^5t^9 + 1976699347728s^4t^{10}\\
&+ 1605475058688s^3t^{11} + 874662022656s^2t^{12}\\
&+ 285993566208st^{13} + 42361159680t^{14})\\
&+ M_c^{12}(6597472s^{14} + 338366703s^{13}t\\
&+ 7696397325s^{12}t^2 + 99678452662s^{11}t^3\\
&+ 833912155244s^{10}t^4\\
&+ 4841060642370s^9t^5 + 20335964523054s^8t^6\\
&+ 63209010354088s^7t^7 + 146358554142828s^6t^8\\
&+ 250907906992024s^5t^9 + 312764014124112s^4t^{10}\\
&+ 274363712128512s^3t^{11} + 159957170755200s^2t^{12}\\
&+ 55477289484288st^{13} + 8641676574720t^{14})\\
&- 2M_c^6 t(s+t)\\
&\times (56096s^{15} + 2136820s^{14}t + 61039254s^{13}t^2\\
&+ 1006276056s^{12}t^3 + 9868826220s^{11}t^4\\
&+ 63875343113s^{10}t^5 + 292621754603s^9t^6\\
&+ 988998062953s^8t^7\\
&+ 2518818379737s^7t^8 + 4859851351942s^6t^9\\
&+ 7049500248014s^5t^{10} + 7539507992112s^4t^{11}\\
&+ 5747250009888s^3t^{12} + 2946149978112s^2t^{13}\\
&+ 908354248704st^{14} + 127083479040t^{15})\\
&- M_c^{10}(657624s^{15} + 39367890s^{14}t\\
&+ 1104413029s^{13}t^2 + 17344487058s^{12}t^3\\
&+ 170832670892s^{11}t^4\\
&+ 1146876637852s^{10}t^5 + 5529671411978s^9t^6\\
&+ 19757278335172s^8t^7 + 53132548989137s^7t^8\\
&+ 107815796091060s^6t^9 + 163659426240484s^5t^{10}\\
&+ 182253522957504s^4t^{11} + 143985271325760s^3t^{12}\\
&+ 76165483016448s^2t^{13} + 24134155960320st^{14}\\
&+ 3456670629888t^{15}) + M_c^8(33792s^{16} + 2774244s^{15}t\\
&+ 97694544s^{14}t^2 + 1994311862s^{13}t^3\\
&+ 24356767073s^{12}t^4 + 194789687794s^{11}t^5\\
&+ 1093516581704s^{10}t^6\\
&+ 4504581919900s^9t^7 + 13974266124379s^8t^8\\
&+ 33029076981316s^7t^9 + 59471161006872s^6t^{10}\\
&+ 80753245374608s^5t^{11} + 81030643128336s^4t^{12}\\
&+ 58075814264832s^3t^{13} + 28049364706176s^2t^{14}\\
&+ 8164110237696st^{15} + 1080209571840t^{16})\Big], \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&F_1^{(1)}[AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}](s,t)\\
&= -\frac{262144}{27M_c^5 s^7 \left(M_c^2 - t\right)^5 \left(M_c^2 - s - t\right)^5}\\
&\times \Big[1728M_c^{30} - 144M_c^{28}(1393s + 144t) + 144M_c^{26}\\
&\times (7517s^2 + 16668st + 792t^2) - 18M_c^{24}(130927s^3\\
&+ 658936s^2t + 718992st^2 + 21120t^3) + 6M_c^{22}\\
&\times (414903s^4 + 3896270s^3t + 9595884s^2t^2\\
&+ 6850464st^3\\
&+ 142560t^4) - 2M_c^{20}(570966s^5 + 10935641s^4t\\
&+ 49831179s^3t^2\\
&+ 80012160s^2t^3 + 41813496st^4\\
&+ 684288t^5) + 2M_c^{18}(-70283s^6 + 3939119s^5t\\
&+ 38212583s^4t^2\\
&+ 114578976s^3t^3 + 134518860s^2t^4\\
&+ 54136512st^5 + 798336t^6) + M_c^{16}(514957s^7\\
&+ 2926870s^6t - 11814356s^5t^2 - 114600008s^4t^3\\
&- 268330620s^3t^4 - 242157600s^2t^5 - 73652544st^6\\
&- 1368576t^7) - M_c^{14}(361295s^8 + 4828723s^7t\\
&+ 24570531s^6t^2 + 51080312s^5t^3 + 31705580s^4t^4\\
&- 8315448s^3t^5 + 8718192s^2t^6 + 17635968st^7\\
&- 855360t^8) + 8st^3(s+t)^3(8s^8 + 25s^7t + 2536s^6t^2\\
&+ 21366s^5t^3 + 78759s^4t^4 + 157896s^3t^5\\
&+ 179640s^2t^6 + 108864st^7 + 27216t^8) + M_c^{12}\\
&\times (151121s^9 + 2725998s^8t + 22467644s^7t^2\\
&+ 100637416s^6t^3 + 266920660s^5t^4 + 453877540s^4t^5\\
&+ 511993860s^3t^6 + 349531200s^2t^7\\
&+ 104894352st^8 - 380160t^9) - M_c^2 st^2(s+t)^2\\
&\times (320s^9 + 3892s^8t + 110625s^7t^2 + 1057188s^6t^3\\
&+ 4932499s^5t^4 + 13283958s^4t^5 + 21746386s^3t^6\\
&+ 21339360s^2t^7 + 11507112st^8 + 2612736t^9)\\
&- M_c^{10}(42121s^{10} + 891469s^9t + 9756298s^8t^2\\
&+ 61315158s^7t^3 + 234557673s^6t^4 + 572091236s^5t^5\\
&+ 907966580s^4t^6 + 913025712s^3t^7 + 525013056s^2t^8\\
&+ 129865536st^9 - 114048t^{10}) + M_c^8(7984s^{11}\\
&+ 172896s^{10}t + 2390911s^9t^2 + 20059044s^8t^3
\end{aligned}
$$

$$
\begin{aligned}
&+102849261 s^7 t^4+335986166 s^6 t^5+718609852 s^5 t^6\\
&+1005256024 s^4 t^7+884461578 s^3 t^8+441939600 s^2 t^9\\
&+95071968 s t^{10}-20736 t^{11})+2 M_c^4 s t(360 s^{11}\\
&+7056 s^{10} t+133908 s^9 t^2+1462154 s^8 t^3\\
&+8991251 s^7 t^4+34117345 s^6 t^5+84461098 s^5 t^6\\
&+139249436 s^4 t^7+151574975 s^3 t^8+104472969 s^2 t^9\\
&+41237280 s t^{10}+7089912 t^{11})-M_c^6(816 s^{12}\\
&+17956 s^{11} t+301806 s^{10} t^2+3524997 s^9 t^3\\
&+24696371 s^8 t^4+107402983 s^7 t^5+303244845 s^6 t^6\\
&+567265544 s^5 t^7+697527422 s^4 t^8+540613740 s^3 t^9\\
&+238658328 s^2 t^{10}+45603648 s t^{11}-1728 t^{12})\Big],
\end{aligned}
$$

$$
\begin{aligned}
&F_1^{(2)}[AVD_{cc}+AV\bar{D}_{\bar{c}\bar{c}}](s,t) \qquad (29)\\
&=\frac{262144}{81 M_c^5 s^8\left(M_c^2-t\right)^6\left(M_c^2-s-t\right)^6}\\
&\times\Big[20221056 M_c^{36}-576 M_c^{34}(233941 s+561036 t)\\
&+144 M_c^{32}(2753877 s^2+14104544 s t+16667448 t^2)\\
&-4 M_c^{30}(170096945 s^3+1400883912 s^2 t\\
&+3527653392 s t^2\\
&+2738026368 t^3)+24 M_c^{28}(31650883 s^4\\
&+371666283 s^3 t+1502816442 s^2 t^2+2481655488 s t^3\\
&+1422434160 t^4)-M_c^{26}(570679427 s^5\\
&+9029032556 s^4 t+52252923804 s^3 t^2\\
&+138610384704 s^2 t^3\\
&+168906874176 s t^4+76170786048 t^5)\\
&+M_c^{24}(256191463 s^6+5795816012 s^5 t\\
&+46170706992 s^4 t^2\\
&+176966758928 s^3 t^3+347658174288 s^2 t^4\\
&+333497539584 s t^5+122937429888 t^6)-2 M_c^{22}\\
&\times(2466837 s^7+863671542 s^6 t+11311001039 s^5 t^2\\
&+62914230172 s^4 t^3+183012632274 s^3 t^4\\
&+286116482160 s^2 t^5+224984126880 s t^6\\
&+69253636608 t^7)\\
&+M_c^{20}(-91034264 s^8-813592713 s^7 t\\
&+48326175 s^6 t^2+28800780204 s^5 t^3\\
&+158775180088 s^4 t^4\\
&+405100852344 s^3 t^5+539810088048 s^2 t^6\\
&+356995482624 s t^7\\
&+90987152256 t^8)+M_c^{18}(75448045 s^9\\
&+1332147106 s^8 t+9770119165 s^7 t^2\\
&+38874308682 s^6 t^3+89128173899 s^5 t^4\\
&+110253341580 s^4 t^5\\
&+56052424932 s^3 t^6-1916424576 s^2 t^7\\
&+2804763456 s t^8\\
&+9582693120 t^9)-32 t^4(s+t)^4(104 s^{10}\\
&+349 s^9 t+38895 s^8 t^2+388900 s^7 t^3+1790450 s^6 t^4\\
&+4854528 s^5 t^5+8476608 s^4 t^6+9797760 s^3 t^7\\
&+7348320 s^2 t^8+3265920 s t^9+653184 t^{10})\\
&-M_c^{16}(35021597 s^{10}+833102650 s^9 t\\
&+8603539424 s^8 t^2\\
&+50889644964 s^7 t^3+193786066418 s^6 t^4\\
&+500022958888 s^5 t^5+890742644832 s^4 t^6\\
&+1097311247136 s^3 t^7+910938066192 s^2 t^8\\
&+463385184768 s t^9+108408972672 t^{10})\\
&+M_c^2 t^3(s+t)^3\\
&\times(17984 s^{11}+225976 s^{10} t+8137786 s^9 t^2\\
&+91342653 s^8 t^3+513344601 s^7 t^4+1753334331 s^6 t^5\\
&+3950202209 s^5 t^6+6069862064 s^4 t^7\\
&+6365607660 s^3 t^8\\
&+4394297088 s^2 t^9+1806167808 s t^{10}\\
&+334430208 t^{11})+2 M_c^{14}(5369598 s^{11}\\
&+159532368 s^{10} t\\
&+2107768049 s^9 t^2+15931331624 s^8 t^3\\
&+77303868220 s^7 t^4+256070555832 s^6 t^5\\
&+594867075166 s^5 t^6\\
&+973395581112 s^4 t^7\\
&+1105034420970 s^3 t^8+832224987792 s^2 t^9\\
&+374238168480 s t^{10}\\
&+75776553216 t^{11})-M_c^4 t^2(s+t)^2\\
&\times(43888 s^{12}+903104 s^{11} t+22877182 s^{10} t^2\\
&+282654857 s^9 t^3+1903612637 s^8 t^4\\
&+8005381771 s^7 t^5\\
&+22622947685 s^6 t^6+44524612444 s^5 t^7\\
&+61674802264 s^4 t^8\\
&+59340363672 s^3 t^9+37899841968 s^2 t^{10}\\
&+14458383360 s t^{11}+2488195584 t^{12})-M_c^{12}\\
&\times(2248090 s^{12}+79574449 s^{11} t+1317350515 s^{10} t^2\\
&+12475000442 s^9 t^3+74917211664 s^8 t^4\\
&+304907732366 s^7 t^5
\end{aligned}
$$

$$
\begin{aligned}
&+ 873255100426 s^6 t^6 \\
&+ 1787109792136 s^5 t^7 + 2607119746632 s^4 t^8 \\
&+ 2654336950408 s^3 t^9 \\
&+ 1794055635504 s^2 t^{10} \\
&+ 722668267008 s t^{11} + 130908645504 t^{12}) + M_c^{10} \\
&\times (308488 s^{13} + 12561878 s^{12} t + 259953855 s^{11} t^2 \\
&+ 3131090454 s^{10} t^3 + 23484836972 s^9 t^4 \\
&+ 117277669268 s^8 t^5 \\
&+ 408772826718 s^7 t^6 \\
&+ 1021374753100 s^6 t^7 + 1847123871971 s^5 t^8 \\
&+ 2403577776876 s^4 t^9 \\
&+ 2196880552908 s^3 t^{10} \\
&+ 1338590478528 s^2 t^{11} + 487521423936 s t^{12} \\
&+ 80079439104 t^{13}) \\
&+ 2 M_c^6 t (27696 s^{14} + 788592 s^{13} t \\
&+ 18122252 s^{12} t^2 + 256127153 s^{11} t^3 + 2159709194 s^{10} t^4 \\
&+ 11773460135 s^9 t^5 + 44215597904 s^8 t^6 \\
&+ 118957864637 s^7 t^7 + 234050783174 s^6 t^8 \\
&+ 338739874305 s^5 t^9 \\
&+ 357552284964 s^4 t^{10} \\
&+ 268091824970 s^3 t^{11} + 135283963248 s^2 t^{12} \\
&+ 41152829472 s t^{13} + 5692280832 t^{14}) - M_c^8 (21360 s^{14} \\
&+ 1150240 s^{13} t + 29400040 s^{12} t^2 + 473008826 s^{11} t^3 \\
&+ 4622824387 s^{10} t^4 + 29030729246 s^9 t^5 \\
&+ 124308726592 s^8 t^6 + 377796208660 s^7 t^7 \\
&+ 833064465237 s^6 t^8 \\
&+ 1342006821284 s^5 t^9 \\
&+ 1567237665232 s^4 t^{10} + 1293304522224 s^3 t^{11} \\
&+ 714916476432 s^2 t^{12} + 237204615168 s t^{13} \\
&+ 35637528960 t^{14})\Big], \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
&F_1^{(3)}[AVD_{cc} + AV\bar{D}_{\bar{c}\bar{c}}](s,t) \\
&= \frac{262144}{243 M_c^5 s^{10} \left(M_c^2 - t\right)^7 \left(M_c^2 - s - t\right)^7} \\
&\times \Big[1741824000 M_c^{44} - 13824 M_c^{42} (1024717 s \\
&+ 2520000 t) \\
&+ 3456 M_c^{40} (15157181 s^2 + 78409048 s t \\
&+ 95243904 t^2) - 768 M_c^{38} (152602537 s^3 \\
&+ 1244415438 s^2 t \\
&+ 3174024582 s t^2 + 2539071360 t^3) \\
&+ 48 M_c^{36} (3709614441 s^4 + 42282452456 s^3 t \\
&+ 169803625392 s^2 t^2 \\
&+ 284772736512 s t^3 + 168755166720 t^4) \\
&- 8 M_c^{34} (24326003617 s^5 + 362802284320 s^4 t \\
&+ 2037040979064 s^3 t^2 \\
&+ 5374375596864 s^2 t^3 \\
&+ 6667854384384 s t^4 + 3121487953920 t^5) + 16 M_c^{32} \\
&\times (9762324075 s^6 + 183890145551 s^5 t \\
&+ 1354940349368 s^4 t^2 + 5002564231056 s^3 t^3 \\
&+ 9760290607872 s^2 t^4 + 9555769117440 s t^5 \\
&+ 3679463854080 t^6) - 4 M_c^{30} (22457637797 s^7 \\
&+ 532549281496 s^6 t + 4996889068172 s^5 t^2 \\
&+ 24325079883568 s^4 t^3 + 66552894119904 s^3 t^4 \\
&+ 102498878252160 s^2 t^5 + 82476996844032 s t^6 \\
&+ 26843207860224 t^7) + M_c^{28} (31939769109 s^8 \\
&+ 1031865139936 s^7 t + 12519019456864 s^6 t^2 \\
&+ 78810576693872 s^5 t^3 + 286787234374848 s^4 t^4 \\
&+ 622093712687616 s^3 t^5 + 788656117759488 s^2 t^6 \\
&+ 536192491511808 s t^7 + 150284156682240 t^8) \\
&- M_c^{26} (318053197 s^9 + 218468720272 s^8 t \\
&+ 4411307171664 s^7 t^2 + 38715857406848 s^6 t^3 \\
&+ 189114078871984 s^5 t^4 + 556628554281920 s^4 t^5 \\
&+ 1004225800991616 s^3 t^6 + 1080262099763712 s^2 t^7 \\
&+ 632454318425088 s t^8 + 154289933844480 t^9) \\
&+ M_c^{24} (-8778960932 s^{10} - 134452810824 s^9 t \\
&- 488149274250 s^8 t^2 + 4001218034320 s^7 t^3 \\
&+ 48976432024272 s^6 t^4 + 230412433439024 s^5 t^5 \\
&+ 608946149725312 s^4 t^6 + 966382124532480 s^3 t^7 \\
&+ 907484756435712 s^2 t^8 + 461490695018496 s t^9 \\
&+ 97128844001280 t^{10}) + M_c^{22} (7078070576 s^{11} \\
&+ 173778399959 s^{10} t + 1874571584019 s^9 t^2 \\
&+ 11483453562160 s^8 t^3 + 43164708110292 s^7 t^4 \\
&+ 100643321821024 s^6 t^5 + 138667494855120 s^5 t^6 \\
&+ 93684817569344 s^4 t^7 + 3332723177088 s^3 t^8 \\
&- 24935540497920 s^2 t^9 + 968161379328 s t^{10} \\
&+ 7021362216960 t^{11}) - 16 t^5 (s+t)^5 (3960 s^{12} \\
&+ 14151 s^{11} t \\
&+ 1702238 s^{10} t^2 + 20378278 s^9 t^3 + 120761039 s^8 t^4 \\
&+ 456128424 s^7 t^5 + 1201156440 s^6 t^6
\end{aligned}
$$

$$
\begin{aligned}
&+ 2269682496s^5t^7 \\
&+ 3068773200s^4t^8 + 2893812480s^3t^9 \\
&+ 1805006592s^2t^{10} \\
&+ 668860416st^{11} + 111476736t^{12}) - M_c^{20} \\
&\times (3405990647s^{12} + 104523726104s^{11}t \\
&+ 1437801582469s^{10}t^2 + 11759612667954s^9t^3 \\
&+ 63654208362929s^8t^4 + 240386484084080s^7t^5 \\
&+ 652169234475584s^6t^6 + 1289432039042896s^5t^7 \\
&+ 1857555696533856s^4t^8 + 1912110070725888s^3t^9 \\
&+ 1336908589885440s^2t^{10} + 568038597894144st^{11} \\
&+ 110118364102656t^{12}) + 4M_c^2t^4(s+t)^4 \\
&(91504s^{13} + 1159496s^{12}t + 51949345s^{11}t^2 \\
&+ 687364992s^{10}t^3 \\
&+ 4769853365s^9t^4 + 21399318662s^8t^5 \\
&+ 67686740542s^7t^6 + 156321971272s^6t^7 \\
&+ 265402870978s^5t^8 \\
&+ 327237185520s^4t^9 + 284374491696s^3t^{10} \\
&+ 164763942912s^2t^{11} + 57042994176st^{12} \\
&+ 8918138880t^{13}) \\
&+ M_c^{18}(1131091631s^{13} + 42014573246s^{12}t \\
&+ 693307252728s^{11}t^2 + 6787879263196s^{10}t^3 \\
&+ 44334703856018s^9t^4 + 204596890047160s^8t^5 \\
&+ 687431049901512s^7t^6 + 1704996711408576s^6t^7 \\
&+ 3124052755731456s^5t^8 + 4176368330021184s^4t^9 \\
&+ 3956371217584896s^3t^{10} + 2511745562866176s^2t^{11} \\
&+ 956202781427712st^{12} + 164551924776960t^{13}) \\
&- M_c^4t^3(s+t)^3(962656s^{14} + 21094520s^{13}t \\
&+ 691108948s^{12}t^2 + 10002353127s^{11}t^3 \\
&+ 80665258449s^{10}t^4 + 426581637437s^9t^5 \\
&+ 1603433166215s^8t^6 \\
&+ 4449490702576s^7t^7 + 9237022428688s^6t^8 \\
&+ 14298644494848s^5t^9 + 16229093115728s^4t^{10} \\
&+ 13078157302656s^3t^{11} + 7067907479808s^2t^{12} \\
&+ 2293062524928st^{13} + 337147453440t^{14}) - 2M_c^{16} \\
&\times (133029237s^{14} + 5947101882s^{13}t \\
&+ 117421358999s^{12}t^2 + 1355188930116s^{11}t^3 \\
&+ 10363630842574s^{10}t^4 \\
&+ 56081817619836s^9t^5 + 222407868023894s^8t^6 \\
&+ 657773336913072s^7t^7 + 1458291508962840s^6t^8 \\
&+ 2410239762599576s^5t^9 + 2920175104142912s^4t^{10} \\
&+ 2512162544509056s^3t^{11} \\
&+ 1449370729677312s^2t^{12} + 501693012043776st^{13} \\
&+ 78585246351360t^{14}) + M_c^6t^2(s+t)^2(1391744s^{15} \\
&+ 43117128s^{14}t + 1279151434s^{13}t^2 \\
&+ 20465640103s^{12}t^3 + 192094141903s^{11}t^4 \\
&+ 1196102161809s^{10}t^5 \\
&+ 5318240639251s^9t^6 + 17564547014872s^8t^7 \\
&+ 43863878016444s^7t^8 + 83088452331520s^6t^9 \\
&+ 118407481083504s^5t^{10} + 124541123134400s^4t^{11} \\
&+ 93507020744832s^3t^{12} + 47299950093312s^2t^{13} \\
&+ 14419185974784st^{14} + 1998499184640t^{15}) \\
&+ M_c^{14}(43621482s^{15} + 2343138771s^{14}t \\
&+ 56199599441s^{13}t^2 + 773612810138s^{12}t^3 \\
&+ 6932647567340s^{11}t^4 + 43593912548178s^{10}t^5 \\
&+ 200838117586542s^9t^6 + 693892463426240s^8t^7 \\
&+ 1816416867876372s^7t^8 + 3603519546818400s^6t^9 \\
&+ 5370130545307120s^5t^{10} + 5898572062338496s^4t^{11} \\
&+ 4619743847396736s^3t^{12} \\
&+ 2435030828186112s^2t^{13} + 772528110925824st^{14} \\
&+ 111261585899520t^{15}) - M_c^{12}(4569808s^{16} \\
&+ 303380716s^{15}t + 9094156671s^{14}t^2 \\
&+ 154232294898s^{13}t^3 + 1654322273320s^{12}t^4 \\
&+ 12195176483488s^{11}t^5 + 65211887760950s^{10}t^6 \\
&+ 261153355185708s^9t^7 + 796490061350461s^8t^8 \\
&+ 1861164352695808s^7t^9 + 3323329278341024s^6t^{10} \\
&+ 4486352440347568s^5t^{11} \\
&+ 4487438605163072s^4t^{12} + 3215047033035264s^3t^{13} \\
&+ 1556602532015616s^2t^{14} \\
&+ 455363166265344st^{15} + 60694275317760t^{16}) \\
&- 2M_c^8t(473008s^{17} + 22997364s^{16}t + 723877564s^{15}t^2 \\
&+ 13517149022s^{14}t^3 + 154742601512s^{13}t^4 \\
&+ 1189403439533s^{12}t^5 + 6565310531804s^{11}t^6 \\
&+ 27133825223148s^{10}t^7 + 85972286205904s^9t^8 \\
&+ 211182065007357s^8t^9 + 403046816578584s^7t^{10} \\
&+ 594612672095720s^6t^{11} + 669840935465464s^5t^{12} \\
&+ 564179282090816s^4t^{13} + 343245181246848s^3t^{14} \\
&+ 142249221368640s^2t^{15} + 35889377398272st^{16} \\
&+ 4155365007360t^{17}) + M_c^{10}(231552s^{17} \\
&+ 23087576s^{16}t + 898504584s^{15}t^2
\end{aligned}
$$

$$+ 19979428298s^{14}t^3 + 269237598605s^{13}t^4$$
$$+ 2395515473146s^{12}t^5$$
$$+ 15079912639380s^{11}t^6 + 70246320893068s^{10}t^7$$
$$+ 248576830505007s^9t^8 + 677016721065728s^8t^9$$
$$+ 1424186961493664s^7t^{10} + 2304170990875904s^6t^{11}$$
$$+ 2833619756443696s^5t^{12}$$
$$+ 2594347606941888s^4t^{13} + 1708831468731264s^3t^{14}$$
$$+ 763746183277056s^2t^{15}$$
$$+207045959569920st^{16} + 25667685679104t^{17})\Big]. \quad (31)$$

References

1. G.T. Bodwin, E. Braaten, G.P. Lepage, Phys. Rev. D **51**, 1125 (1995)
2. K. Abe, (Belle Collaboration) et al., Phys. Rev. D **70**, 071102 (2004)
3. B. Aubert, (BABAR Collaboration) et al., Phys. Rev. D **72**, 031101 (2005)
4. N. Brambilla, S. Eidelman, B.K. Heltsley et al., Eur. Phys. J. C **71**, 1534 (2011)
5. G.V. Pakhlova, P.N. Pakhlov, S.I. Eidelman, Physics-Uspekhi **53**, 219 (2010)
6. G.T. Bodwin, H.S. Chung, D. Kang, J. Lee, C. Yu, Phys. Rev. D **77**, 094017 (2008)
7. G.T. Bodwin, J. Lee, C. Yu, Phys. Rev. D **77**, 094018 (2008)
8. G.T. Bodwin, D. Kang, J. Lee, Phys. Rev. D **74**, 014014 (2006)
9. K.-Y. Liu, Z.-G. He, K.-T. Chao, Phys. Rev. D **77**, 014002 (2008)
10. Y.-J. Zhang, Y.-Q. Ma, K.-T. Chao, Phys. Rev. D **78**, 054006 (2008)
11. Y.-J. Zhang, Y.-J. Gao, K.-T. Chao, Phys. Rev. Lett. **96**, 092001 (2006)
12. A.E. Bondar, V.L. Chernyak, Phys. Lett. B **612**, 215 (2005)
13. V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005)
14. D. Ebert, A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006)
15. D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Lett. B **672**, 264 (2009)
16. D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Rev. D **70**, 014018 (2004)
17. A.P. Martynenko, Phys. Rev. D **72**, 074022 (2005)
18. E.N. Elekina, A.P. Martynenko, Phys. Rev. D **81**, 054006 (2010)
19. A.P. Martynenko, A.M. Trunin, PoS(QFTHEP2011), 051 (2011)
20. A.P. Martynenko, A.M. Trunin, Phys. Atom. Nucl. **77**, 777 (2014)
21. R. Aaij et al., LHCb Collaboration, Phys. Lett. B **707**, 52 (2012)
22. V.V. Kiselev, A.K. Likhoded, S.R. Slabospitsky, A.V. Tkabladze, Sov. J. Nucl. Phys. **49**, 682 (1989)
23. A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A. Novoselov, Phys. Rev. D **84**, 094023 (2011)
24. A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A. Novoselov, Phys. Rev. D **86**, 034017 (2012)
25. C.-F. Qiao, L.-P. Sun, P. Sun, J. Phys. G **37**, 075019 (2010)
26. P. Ko, J. Lee, C. Yu, JHEP01 **070**, 1 (2011)
27. A.P. Martynenko, A.M. Trunin, Phys. Rev. D **86**, 094003 (2012)
28. A.P. Martynenko, A.M. Trunin, Phys. Lett. B **723**, 132 (2013)
29. Yad. Fiz. 76, suppl. number, 155 (2013)
30. Y.-J. Li, G.-Z. Xu, a K.-Y. Liu and Yu-J. Zhang, JHEP07 **051**, 1 (2013)
31. S.P. Baranov, Phys. Rev. D **84**, 054012 (2011)
32. S.P. Baranov, A.M. Snigirev, N.P. Zotov, Phys. Lett. B **705**, 116 (2011)
33. M. Anselmino, E. Predazzi, S. Ekelin et al., Rev. Mod. Phys. **65**, 1199 (1993)
34. J.G. Körner, M. Krämer, D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994)
35. V.V. Kiselev, A.K. Likhoded, Phys. Usp. **45**, 455 (2002)
36. D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, V.A. Saleev, Z. Physik C **76**, 111 (1997)
37. A.P. Martynenko, Phys. Lett. B **663**, 317 (2008)
38. A.P. Martynenko, Phys. Atom. Nucl. **70**, 1305 (2007)
39. V.V. Braguta, V.V. Kiselev, A.E. Chalov, Phys. Atom. Nucl. **65**, 1537 (2002)
40. A.P. Martynenko, A.M. Trunin, Phys. Rev. D **89**, 014004 (2014)
41. G. Chen, X.-G. Wu, Z. Sun, Y. Ma, H.-B. Fu, JHEP **12**, 018 (2014)
42. G. Chen, X.-G. Wu, J.-W. Zhang, H.-Y. Han, H.-B. Fu, Phys. Rev. D **89**, 074020 (2014)
43. J. Jiang, X.-G. Wu, Sh.-M. Wang, J.-W. Zhang, Zh.-Y. Fang, Phys. Rev. D **87**, 054027 (2013)
44. R. Aaij et al., LHCb Collaboration, EPJ C **71**, 1645 (2011)
45. E. Braaten, S. Fleming, T.C. Yuan, Annu. Rev. Nucl. Part. Sci. **46**, 197 (1996)
46. M. Krämer, Prog. Part. Nucl. Phys. **47**, 141 (2001)
47. J.-P. Lansberg, Int. J. Mod. Phys. A **21**, 3857 (2006)
48. G.T. Bodwin, X. Garcia i Tormo, J. Lee, Phys. Rev. D **81**, 114005 (2010)
49. G.T. Bodwin, X. Garcia i Tormo, J. Lee. Phys. Rev. D **81**, 114014 (2010)
50. G.T. Bodwin, Int. J. Mod. Phys. Conf. Ser. **2**, 9 (2011)
51. J. Kublbeck, M. Böhm, A. Denner, Comput. Phys. Comm. **60**, 165 (1990)
52. T. Hahn, Comput. Phys. Commun. **140**, 418 (2001)
53. J.A.M. Vermaseren, FORM. arXiv:math-ph/0010025
54. S.J. Brodsky, J.R. Primack, Ann. Phys. **52**, 315 (1969)
55. R.N. Faustov, Ann. Phys. **78**, 176 (1973)
56. S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko, Phys. Rev. D **62**, 054021 (2000)
57. D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Atom. Nucl. **68**, 784 (2005)
58. D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Rev. D **66**, 014008 (2002)
59. D. Ebert, R.N. Faustov, V.O. Galkin, W. Lucha, Phys. Rev. D **76**, 114015 (2007)
60. H.L. Lai, J. Huston, S. Kuhlmann, J. Morfin, F. Olness, J.F. Owens, J. Pumplin, W.K. Tung, Eur. Phys. J. C **12**, 375 (2000)
61. J. Pumplin, D.R. Stump, J. Huston et al., JHEP07 **012**, 1 (2002)
62. K. Wang, Y.-Q. Ma, K.-T. Chao, Phys. Rev. D **84**, 034022 (2011)
63. L.-P. Sun, H. Han, K.-T Chao, arXiv:1404.4042
64. J.P. Ma, Z.G. Si, Phys. Lett. B **568**, 135 (2003)
65. W.K. Lai, A.K. Leibovich, arXiv:1410.2091[hep-ph]
66. E. Braaten, M. Kusunoki, Yu. Jia, Th. Mehen, arXiv:hep-ph/0304280